

Scale-Chiral Symmetry and the Sound Velocity in Compact-Star Matter

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When a light scalar dilaton σ and the light-quark vector mesons $V = (\rho, \omega)$ are incorporated into an effective scale-invariant hidden local symmetric (sHLS) Lagrangian, scale symmetry for σ and local gauge symmetry for V , both hidden in QCD in the vacuum, arise as “emergent” symmetries at a density above $n_{1/2} \sim 2n_0$, a phenomenon highly relevant for massive compact stars, hitherto unobserved in standard chiral perturbative approaches. What takes place as the density increases beyond $n_{1/2}$ is (1) a topology change to half-skyrmions, (2) parity doubling in the nucleon structure, (3) the maximum neutron star mass $M \simeq 2.01M_\odot$ and the radius $R \simeq 12.0$ km and (4) the sound velocity $v_s^2/c^2 \simeq 1/3$ due to the ρ meson moving toward the vector manifestation (VM) fixed point $m_\rho \rightarrow 0$ and a precursor to emerging conformal symmetry in dense medium.

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Hidden Symmetries in Nuclear Medium.— There are two baffling observations in nuclear phenomena under extreme conditions. One is the presence of a light scalar meson $\lesssim 600$ MeV and the other is the possible “lightness” of the vector meson ρ which in the matter-free space is heavy. The former has to do with hidden scale symmetry and the latter with hidden local symmetry.

The structure of the light scalar meson listed in the particle data booklet as $f_0(500)$ has been a long-standing puzzle in particle and nuclear physics [1], and is receiving a renewed attention in both low-energy particle/nuclear physics [2–4] and in high-energy physics beyond the Standard Model [5, 6]. It has been proposed [2] that the scalar be interpreted as a Nambu-Goldstone (NG) boson of spontaneously broken scale symmetry that joins the NG boson π in “scale-chiral symmetry” at the IR fixed point. In [2] the deviation from the IR fixed point $\Delta\alpha_s$ is taken to be of $\mathcal{O}(p^2)$ in the scale-chiral counting in addition to the usual chiral counting¹.

The objective of this paper is to suggest that a scale symmetry, though invisible in the QCD vacuum, could show up at a renormalization-group fixed point as an “emergent symmetry” in strongly correlated baryonic matter at high density, in an intricate interplay with a local symmetry hidden in QCD (HLS for short) for the vector mesons [8]. As a strong support to this suggestion we will show that the sound velocity in dense medium approaches $\frac{v_s^2}{c^2} = 1/3$ with an equation of state that gives the maximum mass of compact star $M \simeq 2.01M_\odot$ and radius $R = 11.97$ km. For this purpose we combine both hidden symmetries in the form of scale-invariant HLS

(sHLS for short). A surprising consequence of sHLS observed in [9–11] – which will play an important role in what follows – is that the baryonic matter described with a topology change present in skyrmion description of dense matter valid in the N_c limit reveals parity-doubling in the nucleon structure at $n \gtrsim n_{1/2}$, where dense skyrmion matter goes over to a half-skyrmion matter. The nucleon mass turns out to have two components $m_N = m_0 + \kappa(\langle \bar{q}q \rangle)$ where m_0 is a chirally invariant object and κ is the contribution coming from the spontaneous breaking of chiral symmetry. This result resembles closely the parity-doublet nucleon model in which a chirally invariant mass is present in the Lagrangian *ab initio* [9, 12]. When the quark condensate $\langle \bar{q}q \rangle$ is dialed to zero, the latter vanishes and leaves a large $m_0 \sim 0.8m_N$. Note that m_0 is *not present explicitly*, i.e., emerges in the effective Lagrangian. One may say it gets *revealed* at high density.

Since the hidden local symmetry that brings – in the chiral limit – both ρ and a_1 mesons degenerate with the pions at the VM fixed point has been discussed in detail in the literature [8, 13], below, we will devote more space to scale symmetry, limiting ourselves to two flavors.

Exposing Symmetries.—We introduce a dilaton scalar field σ together with the known NG π ’s in the non-linear representation of scale-chiral symmetry transforming as $\chi = f_\sigma e^{-\sigma/f_\sigma} \rightarrow \lambda\chi$, $U = e^{i\vec{\pi}\cdot\vec{\tau}/f_\pi} \rightarrow g_L U g_R^\dagger$ under scale-chiral transformation $x^\mu \rightarrow \lambda^{-1}x^\mu$ and $g_{L,R} \in [SU(2)_L \times SU(2)_R]_{\text{chiral}}$. We treat the vector mesons $V = (\rho, \omega)$ as local gauge symmetric bosons following the strategy of [8]. With scale-chiral symmetry and the QCD trace anomaly suitably expressed by the low-lying fields in scale-invariant HLS (sHLS), π , V and σ , the Lagrangian has a dilaton potential that accounts entirely for both explicit and spontaneous breaking of scale symmetry signaled by the trace of the energy momentum tensor $\langle \theta_\mu^\mu \rangle \propto B\langle \chi^4 \rangle$ with $\langle \chi \rangle \neq 0$ (in the chiral limit) where

¹ In an alternative approach by [3], the usual chiral counting is supplemented with the large N_c counting in the Veneziano limit with $\Delta n_f = \frac{|N_f - N_f^*|}{N_c}$ taken as $\mathcal{O}(p^2)$. Though different in details, the two share an existence of an IR fixed point and give similar results in our work in dense matter [7]. We shall adopt in this paper the approach of [2].

$B \propto \Delta\alpha_s$ (or Δn_f).² As explained in detail in, e.g., [4, 11], the dilaton condensate $\langle\chi\rangle$ inherits information on the gluon condensate *and the quark condensate* from the matching of the trace of the energy-momentum tensor of sHLS with that of QCD at the matching scale Λ_M . We should stress that *the dilaton condensate is nontrivially connected to the quark condensate*. The renormalization group invariant θ_μ^μ is non-zero, so it may seem problematic to construct an sHLS theory for hadron interactions with a possible IR fixed point. In our formulation working to the leading order in the scale-chiral counting, the problem resides entirely in the quantity B and we shall simply assume it is “small” in Nature even if non-zero. We believe that this assumption is vindicated in our analysis.

Our chief claim is that the scale symmetry could be emergent in medium where the parameters flow to the infrared fixed point with $\langle\chi\rangle = 0$, without β' going to zero. The IR fixed point in [2] is β' going to zero with $\langle\chi\rangle \neq 0$, so our emergent symmetry is different from the IR fixed point in [2].

Dilaton Limit Fixed Point.—It was shown in terms of Wilsonian renormalization group (RG) equation [9, 10] that there is a possible infrared fixed point in sHLS with baryons included (*bsHLS* for short)

$$(m_N, g_{V\rho} - 1, g_A - g_{V\rho}) \rightarrow (0, 0, 0) \quad (1)$$

for the nucleon mass m_N , the ρNN coupling $g_{\rho NN} = (g_{V\rho} - 1)g_\rho$ – with g_ρ the hidden gauge coupling – and g_A for the axial-vector current. This is the dilaton-limit fixed point (DLFP) to be specified below. The parameters of the Lagrangian satisfy a set of coupled renormalization-group equations – that are involved in practice – and can flow in various different directions in a manner analogous to what takes place in HLS without the dilaton [8], where the VM fixed point is reached when the constraints given by QCD for the boundary of the Nambu-Goldstone mode and the Wigner-Weyl mode are imposed.

We consider how this fixed point can be reached in dense medium. Embedded in nuclear medium with the vacuum modified by density, the dilaton field picks up density-dependent condensate $\langle\chi\rangle^*$ resulting from the matching of correlators as in [8] so the “bare masses” of the hadrons in the effective Lagrangian, in medium, carry density-dependent masses [4, 11] scaling in density as

$$\frac{m_N^*}{m_N} \approx \frac{g_V}{g_V^*} \frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \left(\frac{m_\pi^*}{m_\pi}\right)^2 \approx \frac{f_\pi^*}{f_\pi} \approx \frac{\langle\chi\rangle^*}{f_\sigma} \quad (2)$$

at the leading order of the scale-chiral counting in the baryon implemented sHLS (i.e., *bsHLS*) *bare* Lagrangian, where the ‘*’ represents the density dependence in nuclear medium. Here, we use ‘ \approx ’ to indicate that there

can be small differences between different degrees of freedom depending on the choice of the cutoff from which the decimations are made. Thus the “bare masses” are controlled by three parameters $\langle\chi\rangle^*$, g_ρ^* and g_ω^* , where g_V ’s are the hidden local symmetry gauge couplings for V . It turns out that there is a hitherto unnoticed relation, crucial for the argument developed below, between the scalings of $\langle\chi\rangle^*$ and g_V^* .

The resulting Lagrangian has a Walecka-type form with the scaling masses at the leading order. A crucial point to note here is that there are singular terms when $\langle\chi\rangle^*$ goes to zero

$$\mathcal{L}' = \frac{f_\sigma^2 - f_\pi^2}{\langle\chi\rangle^{*2}} \mathcal{A}(\bar{\sigma}, \bar{\pi}) + \frac{g_A - g_{V\rho}}{\langle\chi\rangle^{*2}} \mathcal{B}(\bar{\sigma}, \bar{\pi}, N) \quad (3)$$

arising from the non-linear terms of N , $\bar{\sigma} \equiv \langle\chi\rangle^* \sigma / f_\sigma$ and $\bar{\pi} \equiv \langle\chi\rangle^* \pi / f_\sigma$. Taking the limit $\langle\chi\rangle^* \rightarrow 0$ corresponds to the DLFP, where $f_\pi = f_\sigma$ and $g_A = g_{V\rho}$ are required to avoid the singularity. In this limit, one obtains a Gell-Mann-Lévy (GML)-type linear sigma model with the degenerate $O(4)$ multiplet for the scalar and pseudo-scalar fields (massless in the chiral limit) [10, 14, 15]. There the ρ mesons are decoupled from the nucleons. As $f_\sigma^* \propto \langle\chi\rangle^* \rightarrow 0$ and $m_0 \propto \langle\chi\rangle^* \rightarrow 0$, one arrives at restored scale-chiral invariance (in the chiral limit). Although we won’t go into high-temperature matter, it is perhaps appropriate to mention that the DLFP behavior of the parameters of *bsHLS* in density near the chiral symmetry restoration we find is consistent with the lattice calculation of quark susceptibility at high temperature in [16]. This point will be discussed in a future publication.

“Walking” Dilaton Condensate.—To see how to go near the DLFP, let us consider the in-medium behavior of the dilaton condensate. It was discovered in [10, 11] that the dilaton condensate drops monotonously as density increases to $n_{1/2} \sim 2n_0$ – where $n_{1/2}$ is the density at which a topology change takes place in the skyrmion description – but stays constant (say, “walks”) above that density as one moves toward the DLFP. This “walking” of $\langle\chi\rangle^*$ is caused by that most of the ω -meson mass is generated by the spontaneous breaking of the scale symmetry, that is, $m_\omega^* \propto \langle\chi\rangle^*$ at the leading order of *bsHLS* theory. What happens is that a contribution from the ω meson of the form $\sim \frac{g_{\omega NN}^2}{m_\omega^2}$ blocks the dropping of $\langle\chi\rangle^*$ in such a way that it stays more or less constant after a certain density [10].

First recall that the ρ meson (as well as the a_1 meson unless otherwise stated), endowed with hidden local symmetry, can be made light, with its mass becoming comparable to the pion mass (vanishing in the chiral limit) at the VM fixed point [8, 13]. This can be understood in terms of the celebrated mass formula known as KSFR relation, $m_\rho^2 = a f_\pi^2 g_\rho^2$. In HLS, this formula holds to *all orders* in loop expansion with corrections coming at $\mathcal{O}(m_\rho^2/\Lambda^2)$ where $\Lambda \sim 1$ GeV is the chiral scale [8]. In the matter-free vacuum, a remarkable agreement with

² We follow the language of [2] from here on.

all experiments is obtained with $a = 2$. Now an analysis using a Wilsonian renormalization strategy shows that matched to QCD, HLS has a fixed point at which $(g_\rho, a) = (0, 1)$ to which the ρ mass flows as $m_\rho \sim g_\rho \rightarrow 0$ as the VM fixed point is approached. If its fixed point is driven by density, then the KSRF relation becomes more accurate with the ρ mass decreasing, and the gauge coupling rapidly drops to zero as density goes beyond $n_{1/2}$.

In contrast, the phenomenological analysis for the long lifetime of the C14 [17] finds that $g_\rho^* \approx g_\rho$ near the saturation density n_0 to have the strength of the tensor force reduced so as to reproduce the acceptable value for the C14 lifetime. Then how can one understand this difference in density dependence of g_ρ^* before and after $n_{1/2}$?

Here is a conceptually simple explanation that comes from our analysis. The nuclear matter at equilibrium can be interpreted as the Landau Fermi-liquid fixed point in the framework of Wilsonian RG [18]. In this approach, the quasiparticle interactions have vanishing beta functions in the large N limit where $N \propto k_F/\bar{\Lambda}$ with $\bar{\Lambda} = \Lambda - k_F$ where Λ is the cutoff for decimation. As long as the vector mesons and the dilaton of $bsHLS$ Lagrangian are heavy compared with the Fermi sea scale, and they can therefore be integrated out to give the marginal four-point quasiparticle interactions, nuclear matter can be considered to be at its Fermi-liquid fixed point [19]. This is because relativistic mean-field theory is more or less equivalent to Landau Fermi-liquid theory [20].

Now let us consider the parameter space of $bsHLS$ on top of the Fermi-liquid fixed point. Approaching the IR fixed point with the scale parameter $\bar{\Lambda} \rightarrow 0$, the parameters of the EFT Lagrangian should scale related to each other so as to make the beta function for the quasiparticle interactions be zero, i.e., $\beta(k_F, \bar{\Lambda}) = 0$, at a given Fermi-momentum k_F . Suppose the density is changed from k_{F1} to k_{F2} . Then certain parameters should change, say the quasiparticle mass as an example, from $m^*(k_{F1}, \bar{\Lambda} = 0)$ to $m^*(k_{F2}, \bar{\Lambda} = 0)$ to preserve $\beta(k_{F1}, \bar{\Lambda} = 0) = \beta(k_{F2}, \bar{\Lambda} = 0) = 0$. This means that the Fermi-liquid fixed point quantities are closely related to each other at given density so that $g_V^*(k_F, \bar{\Lambda} = 0)$ as well as $m^*(k_F, \bar{\Lambda} = 0)$ should be dependent on $\langle\chi\rangle^*$ and k_F to have $\beta(k_F, \bar{\Lambda} = 0) = 0$. Thus in the density regime $n \lesssim n_{1/2}$, the condensate $\langle\chi\rangle^*$ given in the mean field [10], locked to the quark condensate $\langle\bar{q}q\rangle$, decreases as observed in experiments [21]. It seems remarkable that it goes just in the right proportion to maintain $\beta(k_F, \bar{\Lambda} = 0) = 0$ as density increases while keeping the parameters such as $g_{\rho,\omega}^*(k_F, \bar{\Lambda} = 0)$ unscaled as required in the C14 decay [17] and in deeply bound pionic systems [21].

In the density regime $n > n_{1/2}$, however, the dilaton condensate $\langle\chi\rangle^*$ should stay constant as predicted by the theory. This requires, provided the Fermi-liquid structure holds still, that $g_{\rho,\omega}^*(k_F, \bar{\Lambda} = 0)$ scale to preserve $\beta(k_F, \bar{\Lambda} = 0) = 0$ as density increases. This interplay between the dilaton condensate and the vector coupling

for the Fermi-liquid fixed point structure turns out to bring about the change in density dependence of g_ρ^* from $g_\rho^* \approx g_\rho$ near $n = n_0$ to $g_\rho^* \rightarrow 0$ near the VM fixed point.

Predictions of $bsHLS$ Theory on Compact Stars.— In [11], the intrinsic density dependence of the “bare” $bsHLS$ Lagrangian enters into Eq. (2) with the qualitatively consistent scaling of the parameters discussed above. Particularly notable is that with the VM property for the ρ meson taken into account for $n \geq n_{1/2}$, the density dependence of g_ω^* is constrained to decrease drastically differently from that of g_ρ^* . This induces the breakdown of the $U(2)$ symmetry.

From the arguments developed above follow two predictions of the $bsHLS$ theory: The stiffening of the symmetry energy at $n \gtrsim 2n_0$ and the sound velocity $v_s^2/c^2 \simeq 1/3$.

The symmetry energy is dominated by the nuclear tensor force and in the $bsHLS$ approach, the tensor force is entirely given by the one-pion and one- ρ exchanges. The two terms come with opposite sign, so as $\langle\chi\rangle^*$ falls following the pion decay constant at increasing density, the strength of the ρ tensor cancels the π tensor. Consequently the net tensor force strength decreases and goes nearly to zero at $n \approx 2n_0$ for two-nucleon distances of $r \gtrsim 1$ fm. This feature, supported by the C14 dating at $n \approx n_0$, undergoes a drastic change – caused by a topology change in skyrmion matter – at $n = n_{1/2}$ as the HLS coupling g_ρ^* is “forced” to drop rapidly by the walking $\langle\chi\rangle^*$. What this dropping g_ρ^* does is then to strongly suppress the ρ tensor, with the consequence that the tensor force is almost entirely given by the π tensor for $n > n_{1/2}$. This makes the “soft” symmetry energy (decreasing tensor force) change over to a “stiff” symmetry energy (increasing tensor force) at $n_{1/2}$ as one can see in Fig. 4 in [11]. This has the dramatic effect on the compact star mass as shown in detail in [11].

The second prediction of $bsHLS$ theory, a dramatic one at that, is on the sound velocity in compact-star matter and its link to the trace of the energy-momentum tensor.

Consider the in-medium vacuum expectation value of the energy-momentum tensor $\langle\theta_\mu^\mu\rangle = \epsilon(n) - 3P(n)$ expressed in terms of the quantities that figure in the equation of state (EoS) of nuclear matter, i.e., the energy density $\epsilon(n)$ and the pressure $P(n)$. The energy-momentum tensor and the sound velocity are related by

$$\frac{\partial}{\partial n} \langle\theta_\nu^\nu\rangle = \frac{\partial \epsilon(n)}{\partial n} \left(1 - 3 \frac{v_s^2}{c^2}\right), \quad (4)$$

where we used $\frac{v_s^2}{c^2} = \frac{\partial P(n)}{\partial n} / \frac{\partial \epsilon(n)}{\partial n}$ for the sound velocity v_s . Now if $\langle\theta_\mu^\mu\rangle$ is a constant independent of density (as predicted in this theory), then the left-hand side of (4) is zero. The energy-density of compact-star matter has no known extremum, i.e., $\frac{\partial \epsilon(n)}{\partial n} \neq 0$, hence the sound velocity will then become $v_s/c = \sqrt{1/3}$.

Let us first look at the sound velocity in the mean-field approximation with $bsHLS$ Lagrangian. The trace of the energy-momentum tensor in dense matter in the mean-

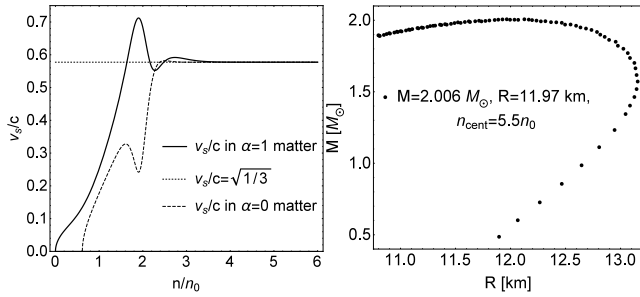


FIG. 1. Left panel: The sound velocity predicted in $bsHLS$ model. The oscillation of v_s near $n_{1/2}$ is considered to be an artifact of the sharp changeover of the states before and after $n_{1/2}$. Here $\alpha = \frac{N-Z}{N+Z}$. Right panel: Mass vs. Radius of the neutron star in beta-equilibrium. With the low-density crust ignored, the low-mass part cannot be trusted.

field approximation with the $bsHLS$ Lagrangian is given by $\langle \theta_\mu^\mu \rangle \propto \langle \chi \rangle^*$ ⁴. This is because to the leading order in the scale-chiral counting, $O(p^2)$, the energy density is scale-invariant since $bsHLS$ is scale-invariant, the scale symmetry breaking being only in the dilaton potential. Therefore if $\langle \chi \rangle^*$ stays constant in $n \gtrsim n_{1/2}$ as discussed above, the sound velocity will become $v_s/c = \sqrt{1/3}$.

Now what happens if one goes beyond the mean-field approximation? Since doing the mean-field approximation with $bsHLS$ corresponds to doing Fermi-liquid fixed theory, going beyond the mean-field is tantamount to calculating higher loop corrections to the Landau fixed point theory. Such a calculation using the V_{lowk} renormalization-group double decimation has been formulated and applied to compact-star matter in [11]. We have applied the same formalism with only a slight modification of the approach to the VM fixed point – which is of course unknown and hence a free parameter – at a higher density than in [11] (without affecting the known properties of nuclear matter well described in [11]). The result is given in Fig. 1 (left panel). That both the $\alpha = 0$ matter and $\alpha = 1$ matter approach $\frac{v_s^2}{c^2} \simeq 1/3$ is consistent with that the $bsHLS$ is scale-invariant to the order $O(p^2)$ in scale-chiral symmetry we are working with.³ The full RG calculation strongly supports the mean-field consideration, implying the high-order correlations beyond the Fermi-liquid fixed point does not change the sound velocity.

We now look at what the EoS that gives the sound velocity $(\frac{v_s}{c})^2 = \frac{1}{3}$ predicts for other properties of compact stars. In Fig. 1 (right panel) is plotted the mass and radius of the star. One obtains the maximum star mass $M_{\max} \simeq 2.01 M_\odot$, the radius $R = 11.97$ km and

the central density $5.5n_0$. Also predicted are the symmetry energy quantities $E_{sym} \approx 26$ MeV, $L \approx 49$ MeV and $K_{sym} \approx -5.1$ MeV. They are generally reasonable.

Concluding Remarks.— With scale symmetry, hidden in the sense described by Yamawaki [6], and hidden local symmetry suitably taken into account, the scale-invariant baryonic HLS Lagrangian with the vacuum sliding with density is shown to lead to $\sim 2M_\odot$ stars in a V_{lowk} renormalization-group approach. The crucial ingredients in the sliding-vacuum properties in the Lagrangian are (1) the topology change at density $n_{1/2} \sim 2n_0$ which plays a key role in the nuclear tensor force that enters dominantly in the symmetry energy and (2) the dropping of the hidden gauge coupling constant (equivalently the ρ mass) going toward the VM fixed point. As a consequence, the sound velocity of the ~ 2 solar-mass star comes out to be $v_s^2/c^2 \approx 1/3$, which is what one would expect for conformally invariant matter. In our calculation, the trace of energy-momentum tensor $\langle \theta_\mu^\mu \rangle$ turns out to be density-independent but is non-zero, so the matter is not conformally invariant in the conventional sense. However the dilaton-limit fixed point is approached when $\langle \chi \rangle \rightarrow 0$, signaling the emergence of scale symmetry.

There is a puzzle in what's happening here. The sound velocity is arrived precisely at $v_s^2/c^2 = 1/3$ by delaying the approach to the VM fixed point from what was done in [11] without, however, changing other parameters. This means that the VM property is intricately tied to the quasi-conformal behavior. In the absence of lattice QCD at high density, there are presently no known theoretical tools to determine how the DL and VM fixed points are reached. It would be extremely challenging and important to devise a method to give model-independent constraints on the parameters involved in the process.

We should say that there is in fact a hint at some sort of scale symmetry in the star matter in that the nuclear tensor force that dominates the symmetry energy – and hence is crucial in the star EoS – is, to the leading order in the chiral-scale counting, scale-invariant and also renormalization-group invariant [23]. Furthermore, that the symmetry energy might encode hidden conformal symmetry at low density is also discussed in [24] in terms of unitary gas.

As a conclusion, we mention an open question that deserves much further studies. Whether the state of matter treated here, endowed with both a possible IR fixed point and the VM fixed point, both of which are not visible in QCD in the vacuum, can be reached in Nature will require a treatment in the half-skyrmion phase which in $sHLS$ sets in for $n > n_{1/2}$, which we believe must overlap with the quarkyonic phase in which quark degrees of freedom are to figure [25].

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³ The EoS used gives normal nuclear matter satisfactorily: Equilibrium density $n_0 = 0.154 \text{ fm}^{-3}$, binding energy $BE = 15.5$ MeV, compression modulus $K = 215.2$ MeV.

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